International Journal of Emerging Technologies in Engineering Research (IJETER)
Volume 2, Issue 3, October (2015) www.ijeter.everscience.org

Special Class of Homo-Cordial Graphs

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Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 1if f (u) =f (v) or 0 if f (u) ≠ f (v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a Homo-Cordial Labeling (HoCL) is called Homo-Cordial Graph (HoCG). In this paper, we proved that the graphs Z-(P_n), Twig Tg_n, ($P_2 \cup mK_1$)+N_2, Jelly Fish J(m,n) are Homo-Cordial Graphs.

Index Terms – Twig, Jelly Fish, Homo-Cordial Graph, Homo-Cordial Labeling, 2000 Mathematics Subject classification 05C78.

1. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e= {uv} of vertices in E is called edges or a line of G. In this paper, we proved that the graphs Z-(P_n), Twig Tg_n, ($P_2 \cup mK_1$)+N_2, Jelly Fish J(m,n) are Homo-Cordial Graphs. For graph theory terminology, we follow [2]

2. PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Homo-Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label 1if f (u) =f (v) or 0 if f (u) ≠ f (v) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

The graph that admits a Homo-Cordial Labeling (HoCL) is called Homo-Cordial Graph (HoCG). In this paper, we proved that the graphs Z-(P_n), Twig Tg_n, ($P_2 \cup mK_1$)+N_2, Jelly Fish J(m,n) are Homo-Cordial Graphs.

Definition: 2.1

In a pair of path P_n, i^{th} vertex of a path P_n is joined with $i+1^{th}$ vertex of a path P_n. It is denoted by Z-(P_n).

Definition: 2.2

A graph obtained from a path by attaching exactly two pendant edges to each internal vertex of the path is called a twig and is denoted by Tg_n , n≥1

Definition: 2.3

The graph ($P_2 \cup mK_1$)+N_2 is a graph with vertex set $\{z_1,z_2,x_1,x_2\ldots x_m\} \cup \{y_1,y_2\}$ and edge

set $\{[(y_1z_1),(y_1z_2),(y_2z_1), (y_2z_2), (z_1z_2)] \cup [(y_1x_i) \cup (y_2x_i) : 1\le i\le m]\}$.

Definition: 2.4

For integers m,n≥0, we consider the graph Jelly Fish J(m,n) with vertex set $V(J(m,n)) = [u,v,x,y],[u_i : 1\le i\le m],[v_i : 1\le i\le n]\}$ and the edge set $E(J(m,n)) = \{[(ux)\cup(uy)\cup(vx)\cup(vy)\cup(xy)]$

$\cup[(uu_i):1\le i\le m]\cup[(vv_i):1\le i\le n]\}$.

3. MAIN RESULTS

Theorem:3.1

Z - P_n is Homo-Cordial Graph.

Proof:

Let $V(Z - P_n) =\{[u_i,v_i : 1\le i\le n]\}$ and

$E(Z - P_n) = \{[(u_iu_{i+1}) : 1\le i\le n-1] \cup [(v_iv_{i+1}) : 1\le i\le n-1] \cup [(v_iu_{i+1}) : 1\le i\le n-1]\}$.

Define $f : V(Z - P_n)\rightarrow\{0,1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 & i \equiv 0,3 \bmod 4 \\ 1 & i \equiv 1,2 \bmod 4 \end{cases} \quad 1\le i\le n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 1,2 \bmod 4 \\ 1 & i \equiv 0,3 \bmod 4 \end{cases} \quad 1\le i\le n$$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1\le i\le n-1$$

$$f^*[(v_iv_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1\le i\le n-1$$

$$f^*[(v_iu_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

Here, $v_f(1) = v_f(0)$ for all n,

$e_f(1) = e_f(0)$ for $n \equiv 1 \bmod 2$ and

$e_f(1) = e_f(0)+1$ for $n \equiv 0 \bmod 2$.

Therefore, Z - P_n satisfies the conditions $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.

Hence, Z - P_n is Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of Z-P_5 and Z-P_6 are shown figure 3.2 and figure 3.3 respectively.

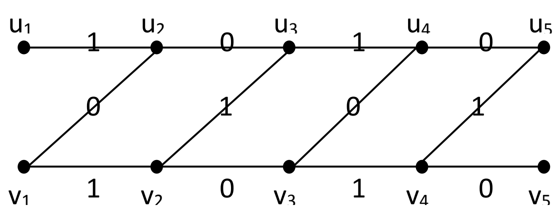

Figure 3.2:

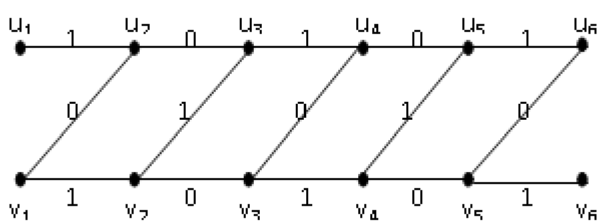

Figure3.3:

Theorem:3.4

Twig Tg_n is Homo-Cordial Graph.

Proof:

Let $V(Tg_n) = \{[u_i: 1\le i\le n], [v_i,w_i : 1\le i\le n-2]\}$ and $E(Tg_n) = \{[(u_iu_{i+1}):1\le i\le n-1] \cup [(u_{i+1}w_i) \cup (u_{i+1}v_i):1\le i\le n-2]\}$

Define $f : V(Tg_n) \rightarrow \{0,1\}$.

The vertex labeling are ,

$$f(u_i) = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-2$$

$$f(w_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-2$$

The induced edge labeling are,

$$f^*[(u_iu_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f^*[(u_{i+1}v_i)] = \begin{cases} 0 & i \equiv 2,3 \bmod 4 \\ 1 & i \equiv 0,1 \bmod 4 \end{cases} \quad 1 \le i \le n-2$$

$$f^*[(u_{i+1}w_i)] = \begin{cases} 0 & i \equiv 0,1 \bmod 4 \\ 1 & i \equiv 2,3 \bmod 4 \end{cases} \quad 1 \le i \le n-2$$

Here, Twig Tg_n satisfies the conditions $|v_f(0)-v_f(1)|\le 1$ and $|e_f 0)-e_f(1)|\le 1$.

Therefore, Twig Tg_n is Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of Tg_5 and Tg_6 are shown in figure 3.5 and figure 3.6 respectively.

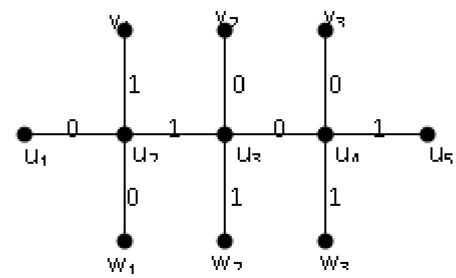

Figure 3.5:

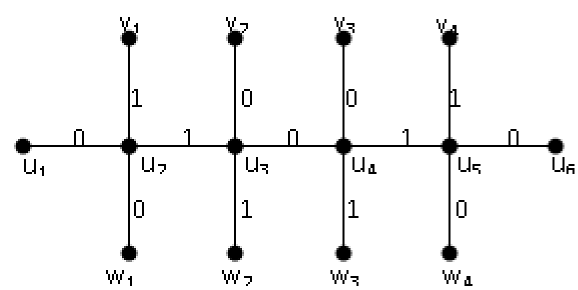

Figure 3.6:

Theorem:3.7

$(P_2 \cup nK_1)+N_2$ is Homo-Cordial Graph.

Proof:

Let G be $(P_2 \cup nK_1)+N_2$.

Let $V(G) = \{[x_i : 1\le i\le n], [y_1,y_2,z_1,z_2]\}$ and

$E(G) = \{[(y_1z_1),(y_1z_2),(y_2z_1), (y_2z_2), (z_1z_2)] \cup [(y_1x_i) \cup (y_2x_i) : 1\le i\le n]\}$.

Define $f:V(G) \rightarrow \{0,1\}$.

The vertex labeling are ,

$f(z_1) = 0$

$f(z_2) = 1$

$f(y_1) = 0$

$f(y_2) = 1$

$f(x_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are,

$f^*[(y_1,z_1)] = 1$

$f^*[(y_1,z_2)] = 0$

$f^*[(y_2,z_1)] = 0$

$f^*[(y_2,z_2)] = 1$

$f^*[(z_1,z_2)] = 0$

$f^*[(y_1x_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

$f^*[(y_2x_i)] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

Here, $v_f(0) = v_f(1)+1$ for $n \equiv 1 \bmod 2$,

$v_f(0) = v_f(1)$ for $n \equiv 0 \bmod 2$ and

$e_f(0) = e_f(1)+1$ for all n.

Therefore, the graph G satisfies the conditions $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$.

Hence, $(P_2 \cup nK_1)+N_2$ is Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of $(P_2 \cup 3K_1)+N_2$ and $(P_2 \cup 4K_1)+N_2$ are shown in figure 3.8 and figure 3.9 respectively.

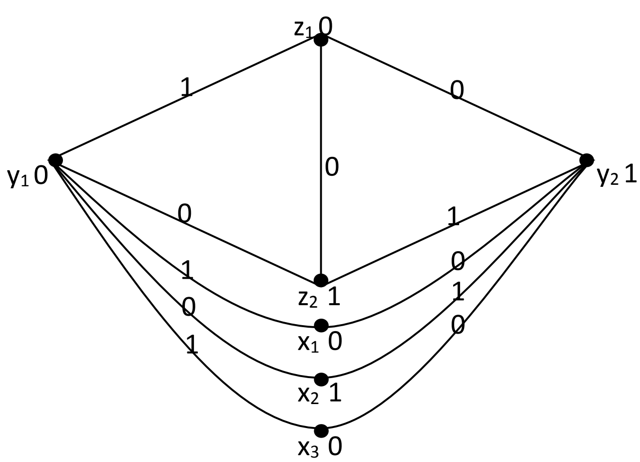

Figure 3.8: $(P_2 \cup 3K_1)+N_2$

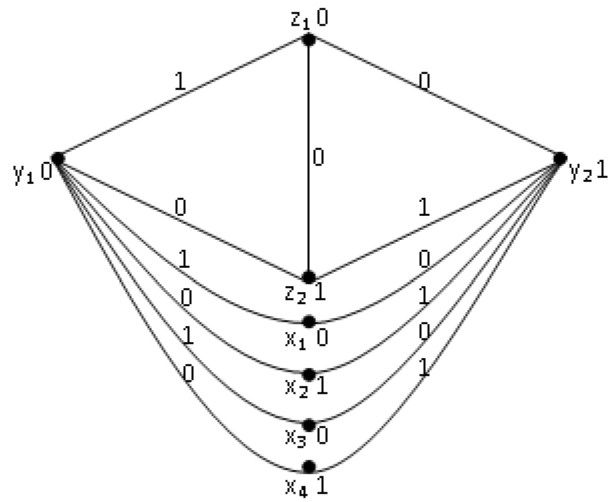

Figure 3.9: $(P_2 \cup K_1)+N_2$

Theorem: 3.10

Jelly Fish $J_{m,n}$ is Homo-Cordial Graph.

Proof:

Let $V(J_{m,n}) = \{[u,v,x,y],[u_i : 1 \leq i \leq m],[v_i : 1 \leq i \leq n]\}$ and $E(J_{m,n}) = \{[(ux) \cup (uy) \cup (vx) \cup (vy) \cup (xy)] \cup [(uu_i): 1 \leq i \leq m] \cup [(vv_i): 1 \leq i \leq n]\}$.

Define $f: V(J_{m,n}) \rightarrow \{0,1\}$.

The vertex labeling are ,

$f(u) = 1$

$f(v) = 0$

$f(x) = 0$

$f(y) = 1$

$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq m$

$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

The induced edge labeling are,

$f^*[(ux)] = 0$

$f^*[(vy)] = 0$

$f^*[(xy)] = 0$

$f^*[(uy)] = 1$

$f^*[(vx)] = 1$

$f^*[(uu_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq m$

$f^*[(vv_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$

Case 1: m-even and n-even

Here, $v_f(0) = v_f(1)$ and $e_f(0) = e_f(1)+1$.

Case 2: m-odd and n-odd

Here, $v_f(0) = v_f(1)$ and $e_f(1) = e_f(0)+1$.

Case 3: m-even and n-odd

Here, $v_f(0) = v_f(1)+1$ and $e_f(0) = e_f(1)$.

Case 4: m-odd and n-even

Here, $v_f(1) = v_f(0)+1$ and $e_f(0) = e_f(1)$.

Therefore, Jelly Fish $J_{m,n}$ satisfies the conditions $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$.

Hence, Jelly Fish $J_{m,n}$ is Homo-Cordial Graph.

For example, the Homo-Cordial Labeling of $J_{2,4}$, $J_{3,5}$, $J_{2,5}$ and $J_{3,4}$ are shown in figure 3.11,

Figure 3.12, figure 3.13 and figure 3.14 respectively.

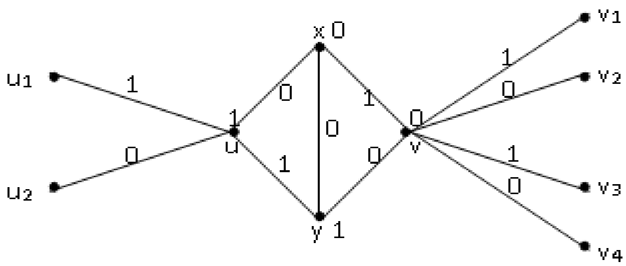

Figure 3.11: $J_{2,4}$

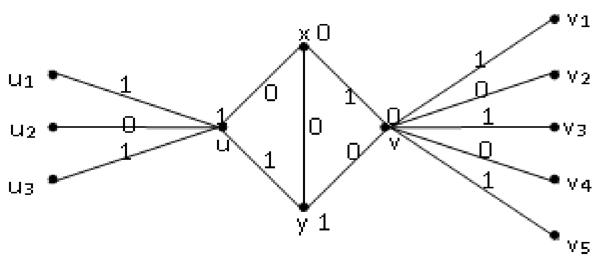

Figure 3.12: $J_{3,5}$

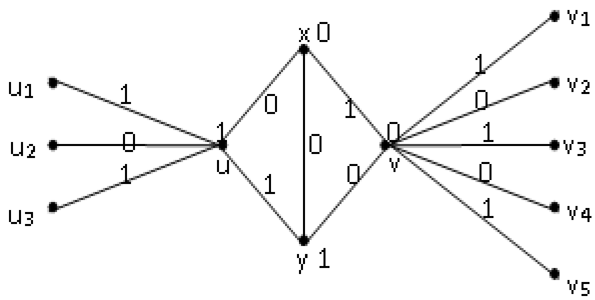

Figure 3.12: $J_{3,5}$

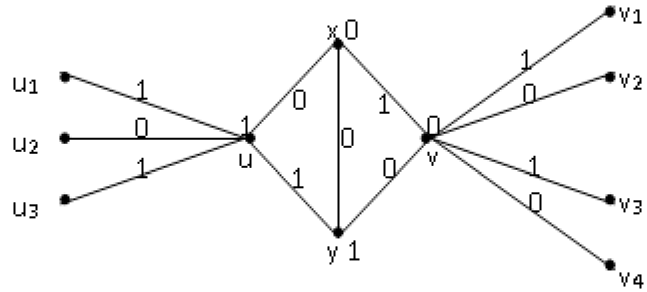

Figure 3.14: $J_{3,4}$

4. CONCLUSION

Homo-Cordial is derived from the concept of Discrete Mathematics, which has wide applications in the field of Digital Technology. It is identified, in this paper, some graphs are satisfying Homo-cordial Labeling. In turn Graph theory has its own applications in Modern Technology. Hence, Homo-Cordial may have a range applications in the Digital World.

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